6. E. A. Artyukhin, "Analysis of sensitivity and experiment planning in problems of identification of processes of generalized heat conduction," in: Heat and Mass Transfer, Pt. VII [in Russian], Vol. 9, ITMO Akad. Nauk BSSR, Minsk (1984), pp. 81-84.
7. A. A. Samarskii, Theory of Difference Schemes [in Russian], Nauka, Moscow (1977).
8. A. I. Boyarinov and V. V. Kafarov, Optimization Methods in Chemical Engineering [in Russian], Khimiya, Moscow (1975).
9. S. B. Stechkin and Yu. N. Subbotin, Splines in Computing Mathematics [in Russian], Nauka, Moscow (1976).
10. A. V. Lykov, Theory of Heat Conduction [in Russian], Vysshaya Shkola, Moscow (1976).

DETERMINATION OF THE THERMOPHYSICAL PROPERTIES OF TRANSLUCENT MATERIALS

P. V. Prosuntsov and S. V. Reznik

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A method is proposed for determining the thermophysical properties of translucent scattering materials in the nonsteady heating regime.

Translucent materials capable of selectively reflecting, transmitting, absorbing, and scattering radiation from external heat sources and background ratiation are in use in a number of thermally loaded structures and are being considered for more such applications. The empirical literature data on the thermophysical properties (TPP) of translucent scatterers in the high-temperature region — where radiative heat transfer is important — is of an approximate nature. Heat transfer occurs in translucent materials simultaneously by conduction and radiation, and the temperature and radiation fields in the materials are coupled. Thus, without isolation of the individual components of heat transfer — conductive and radiative — experimental data on the thermal conductivity and diffusivity of translucent materials cannot be widely used in heat-engineering calculations because they apply only to specific empirical conditions of heat transfer for the given specimen.

The feasibility of using well-known experimental methods of the thermophysics of the optical media [1] to correctly determine TPP and to isolate the individual components of heat transfer in translucent scattering materials is problematic for several reasons. First, mathematical models of inverse coefficient problems of radiative—conductive heat transfer (IPRCT) do not consider such important features of heat transfer as multiple scattering of radiation in absorbing and radiating media. Second, translucent scatterers are generally poor heat conductors. For these materials, as for other thermal insulators, despite the volumetric character of heating it is possible to create small temperature gradients and heating rates in the specimen only in a long experiment employing complicated equipment. We add that determining the optical properties of translucent scatterers at high temperatures is a complicated problem by itself. In this connection, it is important to develop new experimental methods that will make it possible to efficiently determine the TPP of translucent scattering materials in the regime of intensive nonsteady heating.

We will examine a physical and mathematical model of heat transfer in a translucent scattering material for the conditions of stand heat-engineering tests [2].

We will assume that the frontal surface of the plane specimen of isotropic translucent scattering material is heated by a radiation flow of a known spectral composition and density. The coefficient of heat transfer to the gaseous medium on the front surface and the temperature dependence of the optical properties of the material (absorption coefficient α, scattering coefficient β, and refractive index n) are assumed to be known. The rear surface of the specimen is thermally insulated. Experimental thermograms are taken at one or several points of the specimen during heating. It is necessary to determine the temperature dependence of the thermal conductivity and volumetric specific heat of the material.

Heat transfer in a translucent scattering material is described by a system of equations which includes the equations of heat conduction and radiation and the corresponding boundary

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and initial conditions. Since the equation of radiative heat transfer is an integrodifferential equation and involves a great deal of difficulty for its exact solution, we will use the approximate method of moments [3, 4]. Here, we used a binomial approximation of this method which, according to [4], ensures an error in the calculation of all components of the radiation field no greater than 10% with specific scattering $\beta/\alpha > 5$.

Below is the mathematical model of heat transfer in a plane layer of a translucent scattering material after transformation of the equation of radiative transfer and its boundary conditions by the moments method

$$C(T)\frac{\partial T}{\partial \tau} = \frac{\partial}{\partial x}\left(\lambda(T)\frac{\partial T}{\partial x}\right) + q_V; \tag{1}$$

$$q_V = \alpha(T)(M_0 - B^*(T)); \tag{2}$$

$$\frac{d^2M_0}{dx^2} - \frac{1}{\varkappa(T)}\frac{d\varkappa(T)}{dx}\frac{dM_0}{dx} - 3\alpha(T)\varkappa(T)M_0 = -3\alpha(T)\varkappa(T)B^*(T); \tag{3}$$

$$\tau = 0 \quad T(0, x) = T_0(x); \tag{4}$$

$$x = 0 \quad -\lambda(T)\frac{\partial T}{\partial x} = q_T - \alpha_f(T)(T - T_f); \tag{5}$$

$$-\frac{1}{2\varkappa(T)}\left(\frac{1}{3} + R_{11}(T)\right)\frac{dM_0}{dx} + \frac{1}{2}\left(\frac{1}{2} + R_{01}(T)\right)M_0 = q_0(2\eta_m Q_{01}(T) + \eta_\delta Q_1(T)); \tag{6}$$

$$x = h \quad \frac{\partial T}{\partial x} = 0; \tag{7}$$

$$\frac{1}{\varkappa(T)}\left(\frac{1}{3} + R_{11}(T)\right)\frac{dM_0}{dx} + \left(\frac{1}{2} - R(T)\right)M_0 = 0, \tag{8}$$

where $\varkappa(T) = \alpha(T) + (1 - \overline{p})\beta(T)$, $\overline{p}$ is a parameter of the scattering function specified in the Heine–Greenstein form [4]; $M_0 = 2\pi\int_{-1}^{1} I(x, \mu)\,d\mu$ is the zeroth-order radiation moment, which corresponds to the mean radiation intensity; $B^* = 4n^2B(T)$, $B(T)$ is the Planck radiation function; $R_{11}(T)$, $R_{01}(T)$, $Q_{01}(T)$, $Q_1(T)$ are integral reflection and transmission characteristics of the boundary surfaces, the specific form of these characteristics having been given in [4]; η_δ, η_m are the fractions of the directional and diffusive components in the radiant heat flux q_0.

System (1)-(8) can be solved by a numerical method.

The problem of determining the TPP of the translucent material will be solved in an extreme formulation. We will determine the functions $\lambda(T)$ and $C(T)$ which yield the minimum of the functional

$$S = \sum_{j=1}^{M}\sum_{i=1}^{N}(T_p(x_i, \tau_j) - T_e(x_i, \tau_j))^2, \tag{9}$$

where T_p represents theoretical values of temperature found from the solution of system (1-8); T_e are experimental values.

We will approximate the sought dependences by cubic B-splines. For example, we will represent the function $\lambda(T)$ in the form

$$\lambda(T) = \sum_{l=-1}^{m+1}\lambda_l B_l(T);\ B_l(T) = B_0(T - l\Delta T);$$

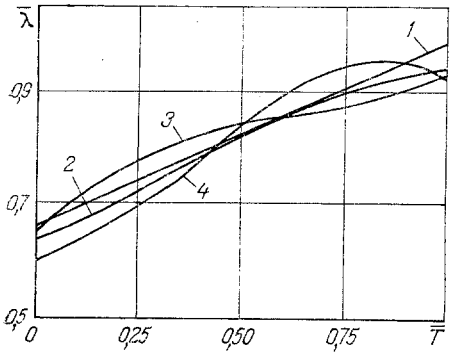

Fig. 1

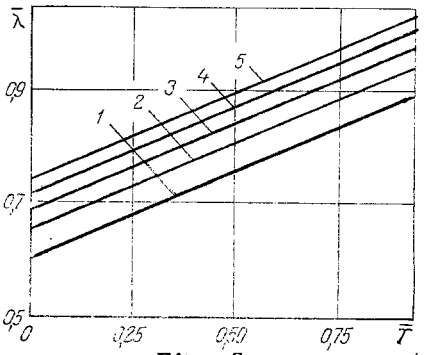

Fig. 2

Fig. 1. Results of determination of the temperature dependence of thermal conductivity for different error distribution laws in experimental thermograms: 1) exact data; 2) saw-toothed law; 3) normal law; 4) uniform law. The amplitude of the temperature perturbation $\overline{\Delta T} = 0.1$.

Fig. 2. Effect of the error of specification of the relative scattering coefficient on the results of determination of $\lambda(T)$: 1) $\bar{\beta} = 0.50$; 2) 0.75; 3) 1.00; 4) 1.25; 5) 1.50.

$$B_0(T) = \frac{1}{\Delta T^3}(\max(0,\ T + 2\Delta T)^3 - 4\max(0,\ T + \Delta T)^3 + 6\max(0,\ T)^3 - 4\max(0,\ T - \Delta T)^3 + \max(0,\ T - 2\Delta T)^3),$$

where ΔT is the step of the spline approximation; m is the number of sections in it.

Thus, the problem of determining the functions $\lambda(T)$ and $C(T)$ reduces to the problem of finding the minimum of a function of $2m + 6$ parameters $S = S(\lambda_{-1}, \lambda_0, \ldots, \lambda_{m+1}, C_{-1}, C_0, \ldots, C_{m+1})$. This problem can be solved by an existing method of optimization such as the method of conjugate gradients [5].

A conjugate problem of the type in [6, 7] is solved to determine the components of the vector of the gradient $\partial S/\partial z_i$, where $z_i = (\lambda_{-1}, \lambda_0, \ldots, \lambda_{m+1}, C_{-1}, C_0, \ldots, C_{m+1})$. Calculation of the components of the vector makes it possible to organize an iterative procedure to find the parameters z_i, $i = 1, 2, \ldots, 2m + 6$. Here, the components of the unit vectors determining the direction of minimization of the function S for the $(k + 1)$-st iteration are found from the relations

$$V_i^{(k+1)} = \frac{-\left(\frac{\partial S}{\partial z_i}\right)^{(k+1)} + \zeta^{(k)} V_i^{(k)}}{\left\{\sum_{j=1}^{2m+6}\left[-\left(\frac{\partial S}{\partial z_j}\right)^{(k+1)} + \zeta^{(k)} V_j^{(k)}\right]^2\right\}^{1/2}},$$

where

$$\zeta^{(k)} = \frac{\left\{\left[\sum_{j=1}^{2m+6}\left(\frac{\partial S}{\partial z_j}\right)^{(k+1)}\right]^2\right\}^{1/2}}{\left\{\left[\sum_{n=1}^{2m+6}\left(\frac{\partial S}{\partial z_n}\right)^{(k)}\right]^2\right\}^{1/2}}.$$

The iteration is stopped upon satisfaction of the condition [8] $S \leqslant \delta^2$, where $\delta^2 = \sum_{j=1}^{M}\sum_{i=1}^{N}[\Delta T_e(x_i, \tau_j)]^2$, $\Delta T_e(x_i, \tau)$, is the error of temperature measurement.

The accuracy and stability of the above-developed method and algorithm for solving a coefficient IPRCT were evaluated by numerical modeling.

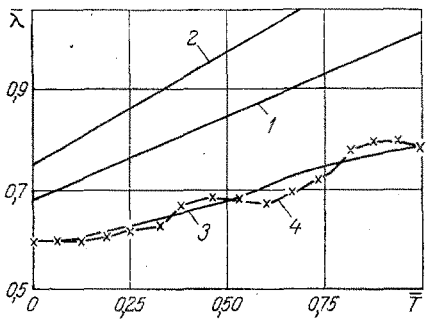

Fig. 3

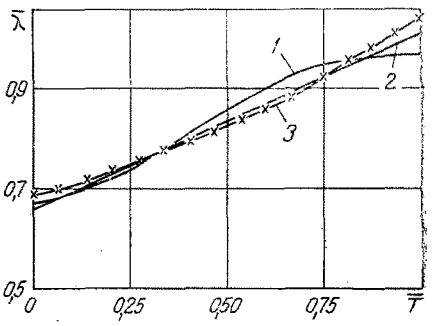

Fig. 4

Fig. 3. Effect of error of specification of the relative absorption coefficient on the determination of $\lambda(T)$: 1) $\bar{\alpha} = 1.00$; 2) 1.25; 3) 0.75, spline approximation; 4) 0.75, tabular representation.

Fig. 4. Effect of the error of the specification of the relative refractive index on the determination of $\lambda(T)$: 1) $\bar{n} = 0.8$; 2) 1.0; 3) 1.2.

A model temperature field was obtained for a material with optical and thermophysical properties close to those of monolithic teflon [9, 10]. The parameters of the thermal loading were as follows: $q_0 = 25 \cdot 10^4$ W/m^2; $q_T = 2.5 \cdot 10^4$ W/m^2; $n_m = 1$; $n_\delta = 0$; $\alpha_f = 5$ W/m·K; $T_f = 293$°K. The boundaries of the specimen, of thickness h = 0.01 m, were assumed to be mirror-reflective. The temperature field was calculated with a time step of 0.1 sec and with a space-coordinate step of $2 \cdot 10^{-4}$ m. The temperature functions C(T) and $\lambda(T)$ were approximated by cubic B-splines with the step $\Delta T = 80$°K, m = 5.

Figures 1-4 show the results of determination of $\lambda(T)$ with the use of five temperature transducers. One of the transducers was located on the front surface of the specimen, while the others were located 0.5, 1.0, 2.0, and 6.0 mm from this surface. The temperature function C(T) was assumed to be known.

It can be seen from Fig. 1 that the algorithm ensures good accuracy and stability with different error distribution laws.

Since the initial data of problem (1)-(9) includes the optical properties of the material, we studied the effect of errors in specifying these properties on the solution of the coefficient IPRCT. It was found that, other conditions being equal, an understatement of the scattering coefficient relative to its nominal value leads to greater errors than does its overstatement (see Fig. 2). This has to do with the fact that, for the conditions examined, the temperature field is less sensitive to an increase in the scattering coefficient.

The absorption coefficient α has an equally strong effect on the solution of the IPRCT. It follows from Fig. 3 that a 25% error in the specification of the absorption coefficient leads to a 20% error in the determination of $\lambda(T)$.

The refractive index n proves to be considerably less important. Even a 20% deviation of its value from the nominal value leads to no more than a 3% error in thermal conductivity (Fig. 4).

The results of the numerical modeling showed that the method places stringent demands on the accuracy of prescription of the components q_0 and q_T of the radiant heat flow. In the case of a 10% error in the prescription of the total heat flux $q_\Sigma = q_0 + q_T$, there may be a 30% in $\lambda(T)$. Here, the error of the prescription of q_T has less of an effect than for the component q_0. This result indicates the need to reliably determine the spectral dependences of the optical properties.

We studied the effect of the number of transducers and the coordinates of their positions on the accuracy of the solution of the IPRCT. With the use of one transducer, the best accuracy for $\lambda(T)$ is attained when the transducer is placed on the front surface. This conclusion applies to both exact and perturbed data. An increase in the number of transducers makes the method more stable with respect to errors in the prescription of the initial data.

NOTATION

$C(T)$, volumetric specific heat; $\lambda(T)$, $\alpha(T)$, $\beta(T)$, thermal conductivity, absorption coefficient, and scattering coefficient, respectively; $n(T)$, refractive index; T, temperature; h, plate thickness; x, coordinate; τ, time; α_f, heat-transfer coefficient; T_f, ambient temperature; q_T, heat flux on the front surface of the specimen on the nontranslucent part of the material; q_0, heat flux on the front surface of the specimen on the translucent part of the material; q_V, power of internal heat sources; $T_0(x)$, initial temperature distribution in specimen; ζ, conjugation parameter; k, number of iteration; ΔT, error of specification experimental values of temperature; $\bar{\alpha} = \alpha/\alpha_m$, relative absorption coefficient; $\bar{\beta} = \beta/\beta_M$, relative scattering coefficient; $\bar{n} = n/n_M$, relative refractive index; $\bar{\lambda} = \lambda/\lambda_{max}$, relative thermal conductivity; $\bar{T} = (T - T_{min})/(T_{max} - T_{min})$, relative temperature; $\varkappa(T)$, attenuation factor; $\Delta\bar{T}$, relative error of specification of experimental values of temperature. Indices: M, model material.

LITERATURE CITED

1. O. A. Sergeev and A. G. Shashkov, Thermophysics of Optical Media [in Russian], Nauka i Tekhnika, Minsk (1983).
2. G. B. Sinyarev, "General principles and organization of stand heat-engineering tests," Nauch. Tr. MVTU, No. 392, 3-16 (1982)..
3. V. V. Gorskii and V. A. Tovstonog, "Application of the moments method to the solution of a radiative transfer equation," Nauch. Tr. MVTU, No. 205, 70-78 (1976).
4. V. N. Eliseev and V. A. Tovstonog, Theoretical Principles of Complex Heat Transfer in Structural Elements [in Russian], Vol. 1, MVTU, Moscow (1982).
5. É. Polak, Numerical Optimization Methods [Russian translation], Mir, Moscow (1974).
6. E. M. Berkovich and A. A. Golubeva, Numerical Solution of Certain Inverse Problems for the Heat-Conduction Equation [in Russian], Moscow State Univ. (1974), pp. 59-75.
7. A. A. Goryachev and V. M. Yudin, "Solution of an inverse coefficient problem of heat conduction," Inzh.-Fiz. Zh., 43, No. 4, 641-648 (1982).
8. O. M. Alifanov, Identification of Heat-Transfer Processes in Aircraft [in Russian], Mashinostroenie, Moscow (1979).
9. V. A. Tovstonog, "Method of combinatorial statistical identification of the optical constants of light-scattering materials," Nauch. Tr. MVTU, No. 392, 47-54 (1982).
10. S. V. Reznik, "Experimental–theoretical determination of the thermophysical properties of thermal insulating materials," Nauch. Tr. MVTU, No. 392, 55-62 (1982).